Dividing with Exponent Rules

Math 97 Supplement 3

LEARNING OBJECTIVES

1. Simplify expressions that involve a monomial divided by a monomial.

Reducing Fractions

When you simplify a fraction, you can divide out factors from the numerator and denominator. For example, suppose we want to reduce $\dfrac{84}{180}$.

One way of doing this is to list out the factors of each number, then divide out the common factors from the numerator and denominator:

$$\frac{84}{180} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 7}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 3 \cdot 5} = \frac{7}{15}$$

Note that you can only divide out factors that are multiplied, and not terms that are added: $\dfrac{4+9}{4+12} = \dfrac{13}{16}$ which doesn't reduce. So you cannot divide out the 4's or reduce the 9 and 12 because of the addition in the numerator and denominator.

You should notice in this section that we will not have any addition or subtraction in the numerator and denominator of each example.

Reducing Fractions with Variables

When you divide fractions with variables, you can use the same idea as dividing numbers:

$$\frac{x^4}{x^6} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \frac{1}{x^2}$$

$$\frac{x^7}{x^5} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = \frac{x^2}{1} = x^2$$

Notice that you can subtract the exponents to get the result in the final answer. After subtracting the exponents (large exponent – small exponent), make sure to leave the remaining variables in the same position that contained the higher exponent.

Example 1

Simplify the following expressions:

a. $\dfrac{12x^5}{8x^{12}}$

b. $\dfrac{12x^5 y^{12}}{20xy^{15}}$

c. $\dfrac{9x^2}{36x^6}$

Solution:

a. $\dfrac{12x^5}{8x^{12}} = \dfrac{4\cdot 3\cdot \cancel{x^5}}{4\cdot 2\cdot x^{\cancel{12}(7)}} = \dfrac{3}{2x^7}$

b. $\dfrac{12x^5 y^{12}}{20xy^{15}} = \dfrac{4\cdot 3\cdot x^{5(4)}\cdot \cancel{y^{12}}}{4\cdot 5\cdot \cancel{x}\cdot y^{\cancel{15}(3)}} = \dfrac{3x^4}{5y^3}$

c. $\dfrac{9x^2}{36x^6} = \dfrac{\cancel{9}\cdot \cancel{x^2}}{4\cdot \cancel{9}\cdot x^{\cancel{6}(4)}} = \dfrac{1}{4x^4}$

Example 2

Simplify the following expressions:

a. $\left(\dfrac{8x^3}{14x^7}\right)^2$

b. $\left(\dfrac{27x^8}{6x^6}\right)^3$

Solution:

a. $\left(\dfrac{8x^3}{14x^7}\right)^2 = \left(\dfrac{\cancel{2}\cdot 4\cdot \cancel{x^3}}{\cancel{2}\cdot 7\cdot x^{\cancel{7}(4)}}\right)^2 = \left(\dfrac{4}{7x^4}\right)^2 = \dfrac{4^2}{7^2\left(x^4\right)^2} = \dfrac{16}{49x^8}$

b. $\left(\dfrac{27x^8}{6x^6}\right)^3 = \left(\dfrac{\cancel{3}\cdot 9\cdot x^{\cancel{8}(2)}}{\cancel{3}\cdot 2\cdot \cancel{x^6}}\right)^3 = \dfrac{9^3\left(x^2\right)^3}{2^3} = \dfrac{729x^6}{8}$

TOPIC EXERCISES

Divide and Simplify.

1. $\dfrac{14x^3}{2x}$

2. $\dfrac{35x^{12}}{45x^7}$

3. $\dfrac{25m^5}{75m^6}$

4. $\dfrac{8x}{4x^5}$

5. $\dfrac{26x^9}{6x^7}$

6. $\dfrac{12c^3}{90c^4}$

7. $\dfrac{14x^3 y}{16xy^3}$

8. $-\dfrac{10x^2 y^8}{15x^6 y^5}$

9. $-\dfrac{4x^3 y^5}{18x^3 y^2}$

10. $\dfrac{33x^6 y^9}{22x^4 y^8}$

11. $\dfrac{3x^5 y}{30x^6 y^8}$

12. $\dfrac{7x^4 y^2}{49xy}$

13. $\dfrac{(7x^3)(6x^8)}{14x^5}$

14. $\dfrac{(4x^2)(9x^5)}{12x^{10}}$

15. $\dfrac{(6x)(15x^7)}{9x^2}$

16. $\dfrac{(16x^7)(3x^{11})}{40x^{20}}$

17. $\left(\dfrac{8x^4}{2x^6}\right)^5$

18. $\left(\dfrac{7x^5}{21x^2}\right)^3$

19. $\left(\dfrac{20x^9}{15x^6}\right)^2$

20. $\left(\dfrac{5x^{12}}{40x^2}\right)^2$

21. $\left(\dfrac{16x^7}{40x^6}\right)^4$

ANSWERS

1. $7x^2$

3. $\dfrac{1}{3m}$

5. $\dfrac{13x^2}{3}$

7. $\dfrac{7x^2}{8y^2}$

9. $-\dfrac{2y^3}{9}$

11. $\dfrac{1}{10xy^7}$

13. $3x^6$

15. $10x^6$

17. $\dfrac{1024}{x^{10}}$

19. $\dfrac{16x^6}{9}$

21. $\dfrac{16x^4}{625}$