ALGEBRA LAWS:
Commutative, Associative, Distributive Laws

Why does (3 + 4) + 10 give you the same answer as (4 + 3) + 10 ?

5(2 + 6) will produce the same result as 5(2) + 5(6), but why?

There are many laws which govern the order in which you perform operations in arithmetic and in algebra. The three most widely discussed are the Commutative, Associative, and Distributive Laws.

The Commutative Law
("change" the order of the numbers or letters)

Over the years, people have found that when we add or multiply, the order of the numbers will not affect the outcome.

5 + 4 is 9 and 4 + 5 is 9. The 5 and the 4 can be switched in addition.
9(2) is 18 and 2(9) is 18. The 9 and the 2 can be switched in multiplication.

"Switching" or "changing" the order of numbers is called "commuting".
People "commute" from home to work everyday. You travel from where you live to where you work, and at the end of the day *turn around* to return to where you live.

When we change the order of the numbers, we have applied the "Commutative Law".
In an addition problem, it is referred to as the "Commutative Law of Addition".
In multiplication, it is the "Commutative Law of Multiplication."

The Commutative Law *does not* work for either subtraction or division.
The order of the numbers *will* affect the outcome.

When you write 100 - 25, it is very different from 25 - 100.

The first case could be likened to writing a $25 check when you have $100 in the bank.
The other case, 25 - 100, would be an example of writing a $100 check when you have $25 in the bank. Oops.....

One must be cautious with subtraction. If we look at the operation of subtraction as "adding the opposite" then there may be a way to "commute" the numbers, but in terms of addition and not subtraction.

5 - 1 is not the same as 1 - 5.

The first case produces positive 4, but the second case yields negative 4.
We can not commute the numbers in subtraction.

Yet, if we think of 5 - 1 as 5 + (-1), five plus the opposite of one, then the *operation* has been changed from subtraction to addition. And we can commute in addition.

5 + (-1) is the same as (-1) + 5. Both "additions" will produce the number positive 4.

KEY IDEA:
We can commute when we add or multiply, but we can not commute or "change" the order of the numbers when we divide or subtract. So (3 + 4) + 10 will give you the same answer as (4 + 3) + 10 because we used the Commutative Law of Addition.

The Associative Law
("the parentheses shift and the numbers do not ").

We have learned to add two numbers at a time, but when we have three or more numbers to add, where do we begin? Does it matter?

This is why the Associative Law was created.

For example, consider 3 + 10 + 2

You could first add 3 and 10 to get 13. Then add the result to 2 and obtain 15.
(3 + 10) + 2 = (13) + 2 = 15

Or you could first add 10 and 2 to get 12. Then add the result to 3 to get 15.
3 + (10 + 2) = 3 + (12) = 15

In both cases, we obtained the same answer.
(3 + 10) + 2 = 3 + (10 + 2)

Notice that the numbers: 3, 10, and 2 did not move.
What *DID* move was the parentheses.
In the first case, the parentheses were associated with the first two numbers 3 and 10.
The second time we tried the problem, they were placed around (associated with) the 10 and 2.

The Associative Law allows you to move parentheses as long as the numbers do not move.
As with the commutative law, this will work only for addition and multiplication. The Associative Law is similar to someone moving among a group of people associating with two different people at a time. You talk to Will and Susan for awhile, then move your attention to Susan and Bob. Bob is next to Anita, so you chat with the two of them.   The people remain

standing in the same place as you turn your head to converse with different people. You associate your attention to two people at a time, but can move your attention and not move the people.

Examples of the Associative Law of Addition:
Look at both sides of the equation in the first step.
Parentheses move, but the numbers stay in the same order.

$$3 \ + \ (\ 7 \ + \ 8\ ) \ = \ (\ 3 \ + \ 7)\ +8$$
$$3 \ + \ (\ \ \ 15\ \ ) \ = \ (\ \ \ 10\ \ )\ +8$$
$$18\ \ \ = \ \ \ 18$$

$$(\ 3x\ + \ 2x\ ) + \ 5x \ = \ 3x \ + \ (\ 2x\ + \ 5x\ )$$
$$(\ \ \ \ 5x\ \ \ ) + \ 5x \ = \ 3x \ + (\ \ \ 7x\ \ \ \ )$$
$$10x \ = \ 10x$$

Examples of the Associative Law of Multiplication:

$$5 * (\ 5 * 6\ ) \ = \ (\ 5 * 5\ ) * 6$$
$$5 * (\ \ 30\ \ ) \ = \ (\ \ 25\ \ ) * 6$$
$$150 \ = \ 150$$

$$2x \ \ * (\ 3y * 4z\ ) \ \ = \ \ (\ 2x * 3y\ ) * 4z$$
$$2x \ \ * (\ \ 12yz\ \ ) \ \ = \ \ (\ \ \ 6xy\ \ ) * 4z$$
$$24xyz \ \ = \ \ 24xyz$$

Why doesn't it work with subtraction?

$$3 - (\ 5 - 2\ ) \ = ?\ (\ 3 - 5\ ) - 2$$
$$3 - (\ \ \ 3\ \ ) \ = ?\ (\ \ -2\ \ )\ - 2$$
$$0\ \ \ \ = ?\ \ -4$$

because it does not produce a correct answer.

How about division?

$$15 \ \ / \ (\ 10 / 2\ ) \ \ = ?\ (\ 15 / 10\ ) / 2$$
$$15 / \ (\ \ \ 5\ \ ) \ \ = ?\ (\ \ 3 / 2\ \ )\ / 2$$
$$3\ \ \ \ = ?\ \ 3/4$$

Nope. It does not work with division.

KEY IDEA:
In the Associative Law, the parentheses move but the numbers or letters do not. The Associative Law works when we add or multiply. It does NOT work when we subtract or divide.

The Distributive Law
("multiply everything inside parentheses by what is outside it").

Think of a delivery truck. It must move from the warehouse to several distributors along its route unloading its merchandise at each business. When the truck has unloaded at one stop, it moves to the next stop, unloads, and moves on until all locations have been visited.   The distributive law is somewhat like a delivery truck, it is distributing multiplication among terms. The truck is outside the parentheses and the businesses are inside separated by plus and minus signs. When we multiply two numbers, each of the numbers is called a factor.

When (5) and (2) are multiplied producing 10, the 5 is one factor and the 2 is another factor.

Now when we multiply (5) * (6 + 2 - 4) the 5 is one factor, but the other factor is an addition and subtraction problem: 6 + 2 - 4. The 6, 2, and 4 are not factors. They are joined together with addition signs and a subtraction sign making them "terms".

We've been told in other math classes to "do inside parentheses first."
Thus in the last example,

        (5) * (6 + 2 - 4)
    = (5) * (8 - 4)
    = (5) * (4)
    = 20

But if we look at the problem in a different light, i.e. multiply each number inside the parentheses by the number outside we would obtain:

        (5) * (6 + 2 - 4)
    = (5) * 6 + (5) * 2 - (5) * 4
    =   30   +   10   -   20
    =   40   - 20
    =   20
The result is the same as above.

This is the idea of the distributive law. When you have parentheses in which there is addition and/or subtraction, and when there is a factor outside of the parentheses, the factor may be distributed to all terms inside the parentheses. Remember "terms" are separated by addition or subtraction signs. In short, multiply every term inside the parentheses by the factor outside it.

Examples of the distributive law:

      9 * (2 - 7 + 8)
= 9 * 2 - 9 * 7 + 9 * 8
= 18 - 63 + 72
= -45 + 72
= 27
checking the answer: 9 * (2 - 7 + 8) = 9 * (3) = 27

$5x (3x^2 + 2x - 4)$
$= 5x (3x^2) + 5x (2x) - 5x(4)$
$= 15x^3 + 10x^2 - 20x$

How to check an algebraic problem.
In this example, the terms inside the parentheses are not alike. We can't add them.
The only way we could check our work would be to make-up a number for x and see if we
obtained the same answer.

Here's how:
Let x = 2 (I just made it up, you could try your own number).
In $5x (3x^2 + 2x - 4)$ replace the x by 2,
$5*2 (3* 2^2 + 2*2 - 4)$
$= 10 (3 * 4 + 4 - 4)$
$= 10 (12 + 4 - 4)$
$= 10 (16 - 4)$
$= 10 (12)$
$= 120$
Next, try the same number in $15x^3 + 10x^2 - 20x$
$15 * 2^3 + 10 * 2^2 - 20 * 2$
$= 15 * 8 + 10 * 4 - 20 * 2$
$= 120 + 40 - 40$
$= 160 - 40$
$= 120$
The answers match, so it appears that the distributive law was done correctly.

KEY IDEA
The distributive law involves a number or variable outside of parentheses (a factor) and
numbers or variables inside parentheses separated by addition and/or subtraction signs (terms).
Multiply every term inside the parentheses by the factor outside it.
Thanks to the distributive 5(2 + 6) will produce the same result as 5(2) + 5(6).